1	2	3	4	5
6	7	8	9	Total

The University of Texas at Austin
Department of Electrical and Computer Engineering

EE w360C — Algorithms — Summer 2013

Final Exam — Saturday 17 August 2013

EID: ________________________

Name: ________________________

Instructions.

- You have 3 hours to complete the exam. The maximum possible score is 120 including 40 marks bonus questions.
- The exam consists of 9 questions and 12 printed pages.
- It is a closed book / closed notes exam. No calculators, laptops, or other devices are allowed.
- Write your answers legibly on the test pages. Use back of test pages for scratch work. Show intermediate answers and process of solving questions.
- If there is any confusion, write down your assumptions and proceed to answer the question.
- In questions where you have to write an algorithm, you can describe it with reference to the algorithms we discussed in class. For example, you can say BFS with some additional operation at one step. Or use the output of topological sort directly etc.

1. Answer briefly. However, a yes/no or a number answer is not enough. You need to give an argument and reason to get any marks. | 20 pts

(a) Given a quadratic algorithm and a linear algorithm and a problem of size 10000, which will take less time?
Solution:

(b) If algorithm A is faster than algorithm B on all instances of a given problem with size less than 1000, can you say $A = O(B)$ or $B = O(A)$?
Solution:

(c) If an algorithm divides the problem into 6 parts of equal size and spends quadratic time on dividing and combining parts, write its recurrence relation.
Solution:

(d) Solve the above recurrence relation using Master's theorem.
Solution:

(e) Argue why $O(\log_2 n) = O(\log_3 n)$.
Solution:

(f) Prove that a graph with an odd cycle cannot be bipartite.
Solution:

(g) Prove that if every node has incoming edges, it cannot be a DAG.
Solution:

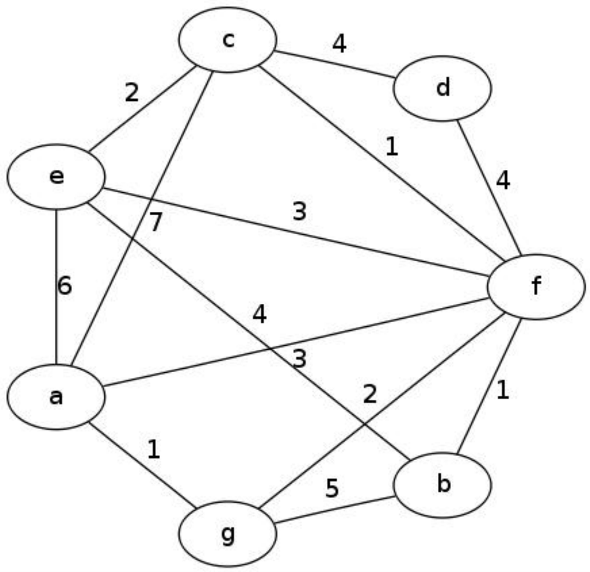
(h) In a dynamic programming algorithm, is the iteration technique better than the memorization technique in terms of time complexity?
Solution:

(i) What is the capacity condition in a network flow?
Solution:

(j) Is the min-capacity cut unique? If not, give a counterexample.
Solution:

2. Find the minimum spanning tree using Prim's algorithm. List the steps taken by the algorithm.

12 pts



Solution:

3. Apply the longest common subsequence algorithm over the words "necessitate" and "essential". Show the dynamic programming matrix M. Trace back the actual longest common subsequence on your matrix.

12 pts

Solution:

4. Suppose you are given an array A with n entries, with each entry holding a distinct number. You are told that the sequence of values $A[1], A[2], \ldots, A[n]$ is *unimodal*: For some index p between 1 and n, the values in the array entries increase up to position p in A and then decrease the remainder of the way until position n. (So if you were to draw a plot with the array position j on the x-axis and the value of the entry $A[j]$ on the y-axis, the plotted points would rise until x-value p, where they'd achieve their maximum, and then fall from there on.)

12 pts

You'd like to find the "peak entry" p without having to read the entire array—in fact, by reading as few entries of A as possible. Show how to find the entry p by reading at most $O(\log n)$ entries of A.

Solution:

5. Assume that all edge costs are distinct in a graph $G = (V, E)$. Let S be any subset of nodes that is neither empty nor equal to all of V, and let edge $e = (v, w)$ be the minimum-cost edge with one end in S and the other in VS. Prove by contradiction that every minimum spanning tree contains the edge e.

12 pts

Solution:

6. Suppose you are managing the construction of billboards on a heavily traveled stretch of road that runs west-east for M miles. The possible sites for billboards are given by numbers $x_1, x_2, \ldots, x_n$, each in the interval $[0, M]$ (specifying their position along the highway, measured in miles from its western end). If you place a billboard at location x_i, you receive a revenue of $r_i > 0$.

12 pts

Regulations imposed by the county's Highway Department require that no two of the billboards be within less than or equal to 5 miles of each other. You'd like to place billboards at a subset of the sites so as to maximize your total revenue, subject to this restriction.

Solution:

7. Bonus: Answer briefly. However, a yes/no or a number answer is not enough. You need to give an argument and reason to get any marks.

16 pts

(a) Show how to solve the decision version of a problem using the optimization version as a blackbox.

Solution:

(b) If problem A can be solved using problem B as a blackbox and B has a polynomial algorithm, what can we say about their relative hardness?

Solution:

(c) If problem A can be solved using problem B as a blackbox, which problem is reduced to which problem and how is it represented?

Solution:

(d) To show two problems A and B are equally hard, we need to reduce them to each other. But if A is NP-Complete, we are done after showing just one reduction. Which reduction is that and why is it enough?

Solution:

(e) If someone solves the vertex cover problem in polynomial time, what can you say about the knapsack problem.
Solution:

(f) Given a polynomial algorithm for the vertex cover problem, is P=NP ?
Solution:

(g) Prove that the decision form of vertex cover problem (is there a vertex cover of size k) is in NP?
Solution:

(h) Consider the decision problem: is there a path between nodes u and v of cost $\leq k$. Is this problem in P, is it NP, is it NP-Complete?
Solution:

8. Bonus: Given ten items and a knapsack of capacity 125, find a way using the dynamic programming algorithm to fill it such that it achieves a value at least as good as the optimal solution for capacity 100. Show the filled matrix M. Your matrix should be as small as possible.

12 pts

Item	Weight	Value
1	73	96
2	46	81
3	53	65
4	61	73
5	34	54

Solution:

12 pts

9. Bonus: Consider the problem of trying to collect baseball cards. Baseball cards come in sealed packets; you don't know what you're getting until you open up the package. Consider packets $P_1, P_2, \ldots P_m$, each of which contains some subset of this year's available baseball cards.

Consider the decision problem that asks whether it is possible to collect all of this year's available cards by buying fewer than k packets of cards. Prove that this problem is NP-Complete using the Vertex Cover Problem (which is known to be NP-Complete).

Solution: