EE 360C — Algorithms — Summer 2013
Homework #6

Due: July 31, 2013 11:30am (in class)

Homework problems are to be done individually. You may discuss the problem and general concepts with other students, but you must write your solutions independently.

Each question is worth 10 points. Maximum possible score is 30.

Whenever you give an algorithm, prove that it is correct.

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1. Suppose it's nearing the end of the semester and you're taking n courses, each with a final project that still has to be done. Each project will be graded on the following scale: It will be assigned an integer number on a scale of 1 to $g > 1$, higher numbers being better grades. Your goal, of course, is to maximize your average grade on the n projects.

   You have a total of $H > n$ hours in which to work on the n projects cumulatively, and you want to decide how to divide up this time. For simplicity, assume H is a positive integer, and you'll spend an integer number of hours on each project. To figure out how best to divide up your time, you've come up with a set of functions $\{f_i : i = 1, 2, \ldots, n\}$ (rough estimates, of course) for each of your n courses; if you spend $h \leq H$ hours on the project for course i, you'll get a grade of $f_i(h)$. (You may assume that the functions f_i are *nondecreasing*: if $h < h'$, then $f_i(h) \leq f_i(h')$.)

   So the problem is: Given these functions $\{f_i\}$, decide how many hours to spend on each project (in integer values only) so that your average grade, as computed according to the f_i, is as large as possible. In order to be efficient, the running time of your algorithm should be polynomial in n, g, and H; none of these quantities should appear as an exponent in your running time.

2. Suppose you are given a directed graph $G = (V, E)$ with costs on the edges c_e for $e \in E$ and a sink t (costs may be negative). Assume that you also have finite values $d(v)$ for $v \in V$. Someone claims that, for each node $v \in V$, the quantity $d(v)$ is the cost of the minimum-cost path from node v to the sink t.

   (a) Give a linear-time algorithm (time $O(m)$ if the graph has m edges) that verifies whether this claim is correct.

   (b) Assume that the distances are correct, and $d(v)$ is finite for all $v \in V$. Now you need to compute distances to a different sink t'. Give an $O(m \log n)$ algorithm for computing distances $d'(v)$ for all nodes $v \in V$ to the sink node t'. (*Hint:* It is useful to consider a new cost function defined as follows: for edge $e = (v, w)$, let $c'_e = c_e - d(v) + d(w)$. Is there a relation between costs of paths for the two different costs c and c'?)

3. The owners of an independently operated gas station are faced with the following situation. They have a large underground tank in which they store gas; the tank can hold up to L gallons at one time. Ordering gas is quite expensive, so they want to order relatively rarely. For each order, they need to pay a fixed price P for delivery in addition to the cost of the gas ordered. However, it costs c to store a gallon of gas for an extra day, so ordering too much ahead increases the storage cost.

   They are planning to close for a week in the winter, and they want their tank to be empty by the time they close. Luckily, based on years of experience, they have accurate projections for how much gas they will need each day until this point in time. Assume that there are n days left until they close, and they need g_i gallons of gas for each of the days $i = 1, \ldots, n$. Assume that the tank is empty at the end of day 0. Give an algorithm to decide on which days they should place orders, and how much to order so as to minimize their total cost.