

EE 360C — Algorithms — Summer 2013

Homework #7

Due: August 7, 2013 11:30am (in class)

Homework problems are to be done individually. You may discuss the problem and general concepts with other students, but you must write your solutions independently.

Each question is worth 10 points. Maximum possible score is 30.

Whenever you give an algorithm, prove that it is correct.

1. Network flow issues come up in dealing with natural disasters and other crises, since major unexpected events often require the movement and evacuation of large numbers of people in a short amount of time.

Consider the following scenario. Due to large-scale flooding in a region, paramedics have identified a set of n injured people distributed across the region who need to be rushed to hospitals. There are k hospitals in the region, and each of the n people needs to be brought to a hospital that is within a half-hour's driving time of their current location (so different people will have different options for hospitals, depending on where they are right now).

At the same time, one doesn't want to overload any one of the hospitals by sending too many patients its way. The paramedics are in touch by cell phone, and they want to collectively work out whether they can choose a hospital for each of the injured people in such a way that the load on the hospitals is *balanced*: Each hospital receives at most $\lceil n/k \rceil$ people.

Give a polynomial-time algorithm that takes the given information about the people's locations and determines whether this is possible.

2. Suppose you are given a directed graph $G = (V, E)$, with a positive integer capacity c_e on each edge e , a source $s \in V$, and a sink $t \in V$. You are also given a maximum s - t flow in G , defined by a flow value f_e on each edge e . The flow f is *acyclic*: There is no cycle in G on which all edges carry positive flow. The flow f is also integer-valued.

Now suppose we pick a specific edge $e^* \in E$ and reduce its capacity by 1 unit. Show how to find a maximum flow in the resulting capacitated graph in time $O(m + n)$, where m is the number of edges in G and n is the number of nodes.

3. Consider the following problem. You are given a flow network with unit-capacity edges: It consists of a directed graph $G = (V, E)$, a source $s \in V$, and a sink $t \in V$; and $c_e = 1$ for every $e \in E$. You are also given a parameter k .

The goal is to delete k edges so as to reduce the maximum s - t flow in G by as much as possible. In other words, you should find a set of edges $F \subseteq E$ so that $|F| = k$ and the maximum s - t flow in $G' = (V, E \setminus F)$ is as small as possible subject to this.

Give a polynomial-time algorithm to solve this problem.